St Hilda's

ANGLICAN SCHOOL FOR GIRLS

Time:	25 minutes
Total Marks:	25 marks

Year 12 Test 2

Thursday 29th April 2021

Resource Free

ClassPad calculators are Not permitted.

Formulae Sheet is Permitted.

Name:

1. (2 , 2 = 4 marks)

   Differentiate the following with respect to x. Do not simplify.

   a) $3x^2 e^x$

   b) $3e^{2x^3+1}$

2. (2, 2 , 2, 1 = 7 marks)

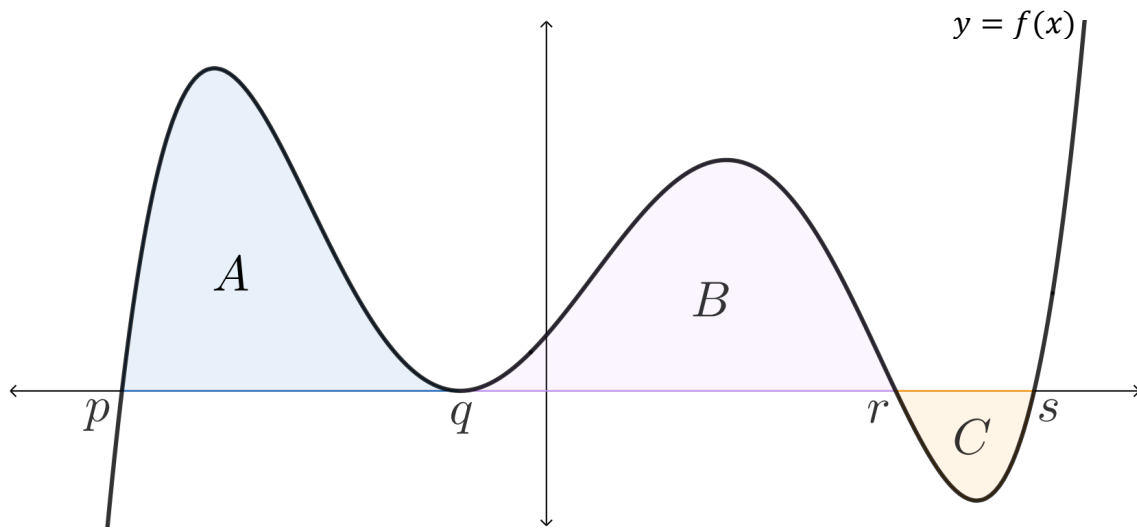a) Evaluate the following $\int \frac{\sqrt{x}+x}{x} dx$.

b) Find Q in terms of p given that $\frac{dQ}{dp} = 4 - \frac{6}{p^3}$ and $Q = -3$ when $p = 1$.

c) $\int 2x^3 e^{x^4} dx$

d) $\frac{d}{dx} \int_{-2}^{x} \frac{t^2+3}{\pi-\sqrt{t}} dt$

3. (1, 1, 2 = 4 marks)

The three regions between the curve $y = f(x)$ and the x-axis have areas of A, B, and C units^2 as shown below. Determine the following definite integrals.
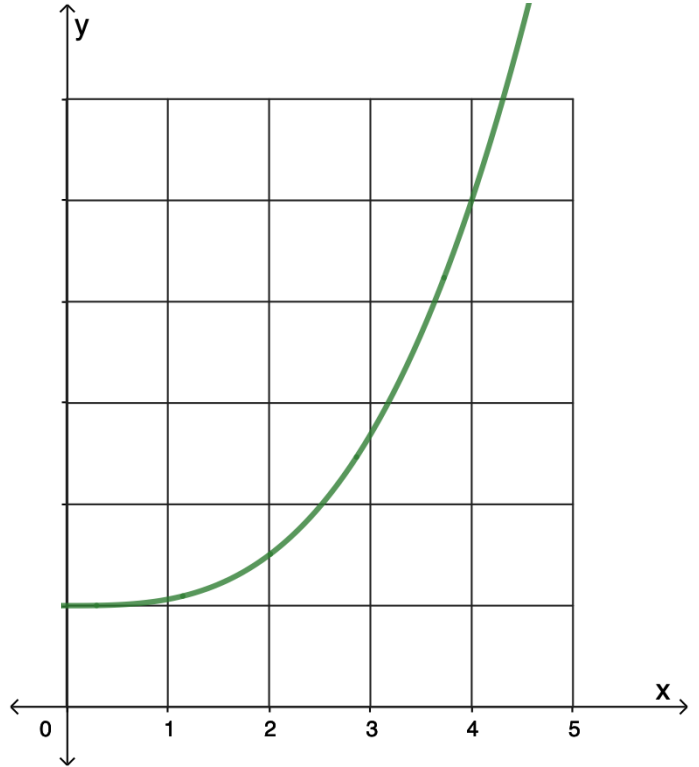

a) $\int_p^s f(x)dx$

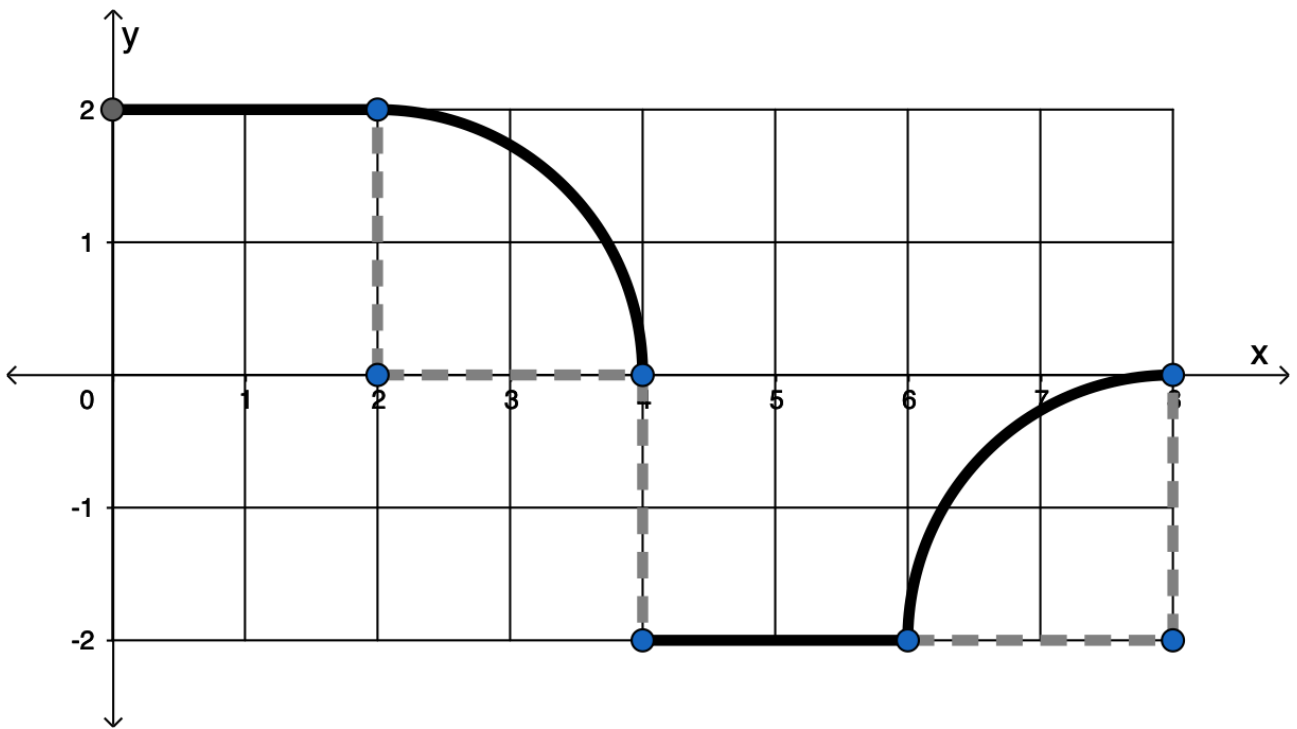b) $\int_p^s 2f(x)dx$

c) $\int_r^p f(x) + 1dx$

4. (2, 2 = 4 marks)

The function $f(x) = x^3 + 1$ is shown below.

a) Using the under-estimate with widths of 1 unit, approximate the area under $f(x)$ for $1 \leq x \leq 3$. Show all working.

b) Combine your under-estimate from part a) and the over-estimate value to get a better approximation for the integral $\int_1^3 x^3 dx$.

5. (2, 2, 2 = 6 marks)

The function $f(x)$ is shown below.

a) Use the graph above to determine the following in exactly.

i. $\int_0^4 f(x)dx$

y

ii. $\int_4^8 f(x)dx$

iii. If $\int_k^8 f(x)dx = 0$, solve for k.

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6. (2, 2 = 4 marks)

An imaginary radioactive isotope Coraronium decays at a rate of $\frac{\mathrm{d}A}{\mathrm{d}t} = -0.14A$ where A (kg) is the amount of Coraronium remaining and t is in years.

a) If 2 kg of Coraronium exists originally, determine how much will remain after 10 years.

b) Determine the half life of Coraronium, that is the time it takes for the radioactive isotope to be reduced to 50%.

7. (1, 2 = 3 marks)

A population changes such that $\frac{dP}{dt} = -0.12P$, where t is in years.

a) Is the the population growing or decaying?

b) If the population is 120 000 after 8 years. Calculate (to the nearest 1000) the original population.

8. (4 marks)

Given $f(x) = e^x$ and $g(x) = e^{-x}$ find the **exact** area of the regions enclosed by the two functions, $x = -1$ and $x = 1$. Show the use of a sketch in your solution.

9. (3, 2, 3, 1 = 9 marks)

A particle's is moving with rectilinear motion and its position can be modelled by the function

$v(t) = 3t^2 - 12t + 9$ for $0 \leq t \leq 4$, where v is measured in metres/seconds and t is measured in seconds.

a) Determine when the velocity of the particle is maximised.

b) If the particle is initially at the origin determine an expression for the displacement.

c) Determine the total distance travelled in the first 3 seconds.

d) Determine the change in displacement in the 2nd second.